Integrals of Trig Functions (7.2):

A: $\int \sin^m x \cos^n x \, dx$

(1) m odd: Strip 1 sine out and convert the rest to cosines using $\sin^2 x = 1 - \cos^2 x$. Then make the substitution $u = \cos x$.

(2) n odd: Strip 1 cosine out and convert the rest to sines using $\cos^2 x = 1 - \sin^2 x$. Then make the substitution $u = \sin x$.

(3) m and n both odd: Use either (1) or (2).

(4) m and n both even: Use double and/or half angle formulas to reduce to a form that can be integrated.

- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin x \cos x = \frac{1}{2}\sin 2x$

B: $\int \tan^m x \sec^n x \, dx$ (and similarly $\int \cot^m x \csc^n x \, dx$)

(1) m odd: Strip 1 tangent and 1 secant out and convert the rest to secants using $\tan^2 x = \sec^2 x - 1$. Then make the substitution $u = \sec x$.

(2) n even: Strip 2 secants out and convert the rest to tangents using $\sec^2 x = \tan^2 x + 1$. Then make the substitution $u = \tan x$.

(3) m odd and n even: Use either (1) or (2).

(4) m even and n odd: Replace $\tan^2 x$ with $\sec^2 x - 1$ and go from there (not an important case)

C: $\int \sin(mx) \cos(nx) \, dx$, $\int \sin(mx) \sin(nx) \, dx$, $\int \cos(mx) \cos(nx) \, dx$

Use the corresponding identity:

- $\sin A \cos B = \frac{1}{2}\big[\sin(A - B) + \sin(A + B)\big]$
- $\sin A \sin B = \frac{1}{2}\big[\cos(A - B) - \cos(A + B)\big]$
- $\cos A \cos B = \frac{1}{2}\big[\cos(A - B) + \cos(A + B)\big]$

Trig substitutions (7.3):

If the integral contains the following expression, make the substitution and use the trig identity given below. Then use right triangle trig to convert back to the original variable.

- $\sqrt{a^2 - b^2x^2} \Rightarrow x = \frac{a}{b}\sin \theta$, $\cos^2\theta = 1 - \sin^2\theta$.
- $\sqrt{a^2 + b^2x^2} \Rightarrow x = \frac{a}{b}\tan \theta$, $\sec^2\theta = 1 + \tan^2\theta$.
- $\sqrt{b^2x^2 - a^2} \Rightarrow x = \frac{a}{b}\sec \theta$, $\tan^2\theta = \sec^2\theta - 1$.